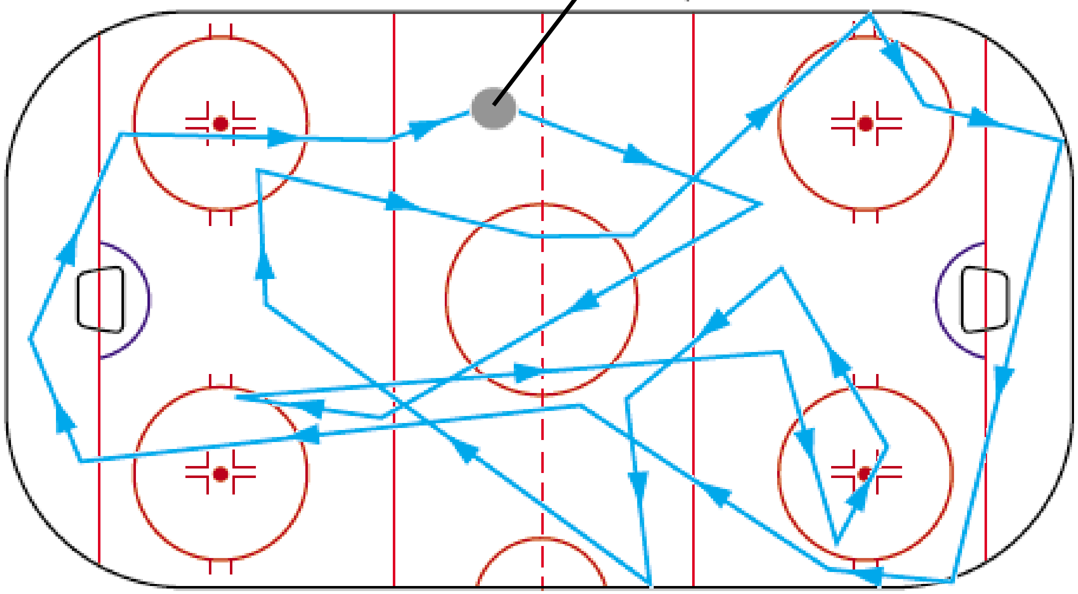
4.4 Hall Effect

F-X
Ext

How fast does the puck move?

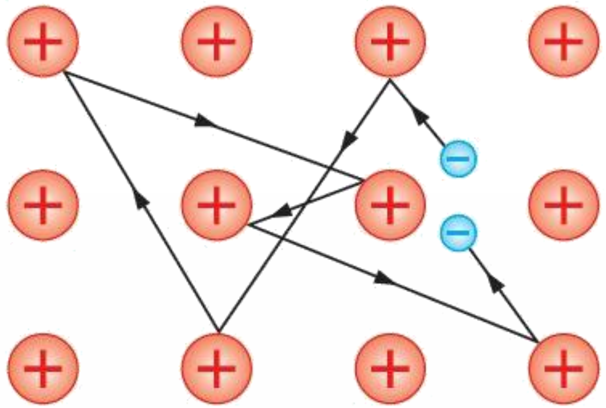
In ice hockey match, the puck bounces around the rink at a speed up to 160 km h^{-1}.



What is the magnitude of the average velocity of the puck over the whole match?

1 Electric current as flow of electric charges

Free electrons in metal move at a very high speed (~10^5 m s^{-1}) and bounce off the stationary +ve ions.

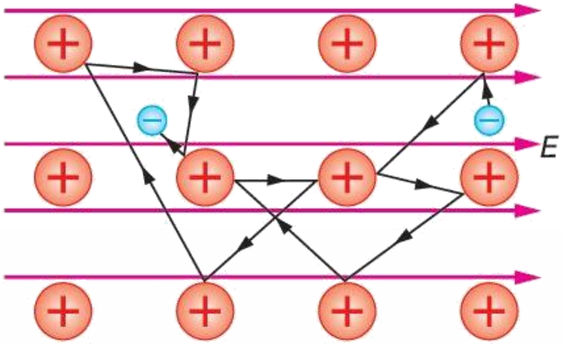


⇒ **random** motion
(like gas molecules)

Like a hockey puck, the total displacement over time is negligible.

∴ no net flow of electrons ⇒ no electric current

1 Electric current as flow of electric charges



Connect the metal to a power supply

⇒ electric field is set up

⇒ electrons experience electric force and accelerate opposite to field direction

⇒ collide with +ve ions and change directions

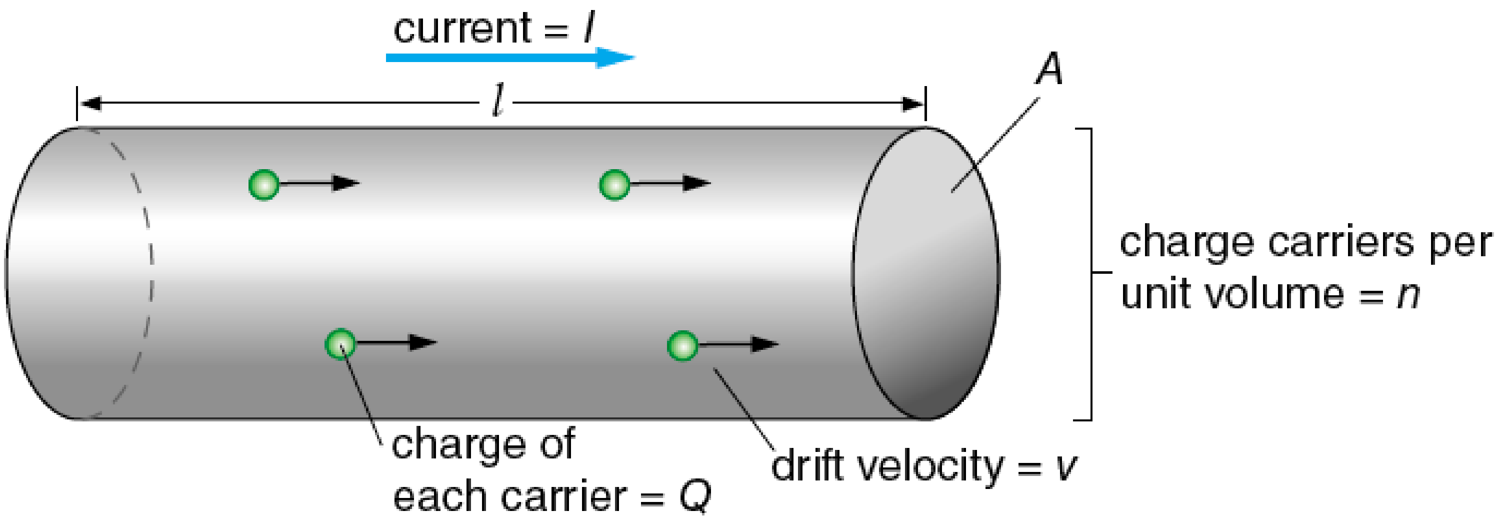
⇒ net displacements ↓

⇒ steady **drift velocity** (~10^{-5} m s^{-1})

Drift velocity of electrons is in opposite direction to the current.

2 Drift velocity and current

Consider the following conductor:

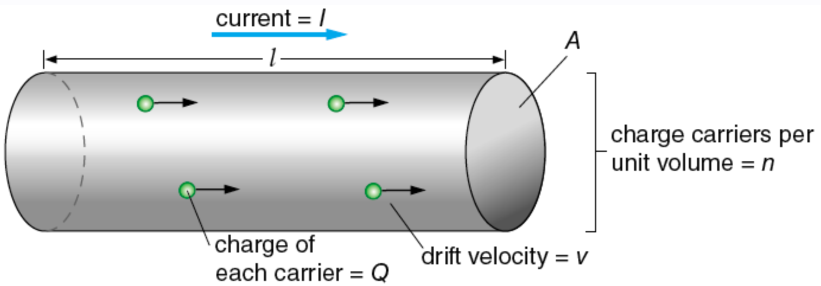


l : length

A : cross-sectional area

2 Drift velocity and current

If p.d. across it makes total charges Q_{total} pass through length l in time t with average drift velocity v, then



current I is given by $I = \dfrac{Q_{total}}{t} = \dfrac{nAlQ}{\frac{l}{v}}$

$$\boxed{I = nAvQ}$$

Example 15 **Nerve conduction**

Check-point 9 – Q1

0.5-A current flows through a copper wire (cross-sectional area = 10^{-7} m^2). If each copper atom contributes 1 free e$^-$, drift velocity of free e$^-$ = ?

(Free e$^-$ per unit volume = 10^{29} m^{-3}, charge on e$^-$ = -1.6×10^{-19} C)

By $I = nAvQ$,

$$v = \frac{I}{nAQ} = \frac{0.5}{(10^{29})(10^{-7})(-1.6 \times 10^{-19})}$$

$$= -3.125 \times 10^{-4} \text{ m s}^{-1}$$

F-X Ext

3 Magnetic force on a moving charge

$\because$ current is a flow of e^-

$\therefore$ magnetic force experienced by current is sum of force acting on each moving charge

Magnetic force acting on a current-carrying conductor is given by

$$F = BIl \sin\theta$$

Combining it with $I = nAvQ$, we have

$$F = B(nAvQ)l \sin\theta$$

3 Magnetic force on a moving charge

Total no. of charge carriers in the conductor

$= nAl$

$\Rightarrow$ magnetic force on each charge carrier F_Q is given by

$$F_Q = \frac{F}{nAl} = \frac{BnAvQl \sin \theta}{nAl}$$

$$= BQv \sin \theta$$

3 Magnetic force on a moving charge

The force acting on charge Q with drift velocity v in a magnetic field B is given by

$$\boxed{F_Q = BQv \sin\theta}$$

Drift velocity $v \perp$ magnetic field B

$\Rightarrow \sin\theta = 1$

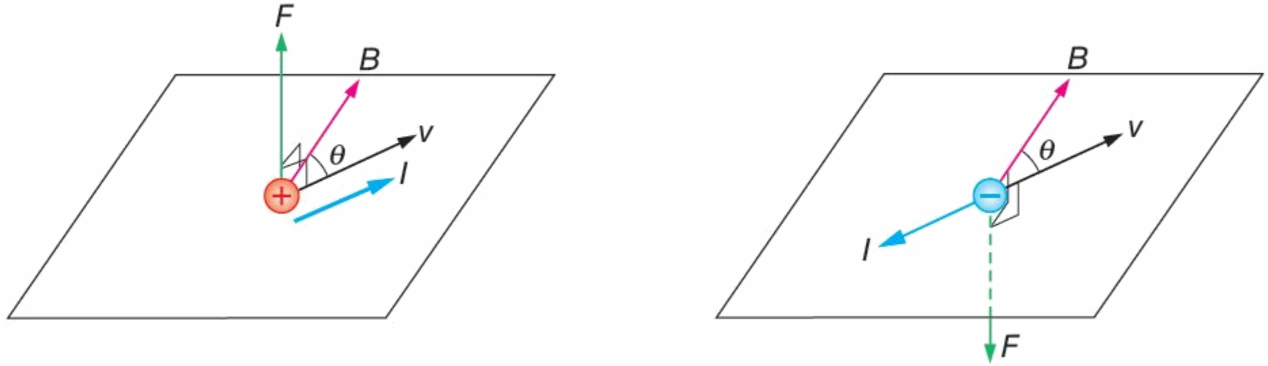
$\Rightarrow$ magnetic force on the charge Q becomes

$$F_Q = BQv$$

3 Magnetic force on a moving charge

Direction of magnetic force can be found by Fleming's left-hand rule.

Current direction is the same as (opposite to) the moving direction of +ve charge (–ve charge).



$\because$ magnetic force $\perp$ moving direction of charge

$\therefore$ no work is done on charge $\Rightarrow$ constant speed

Example 16 **An electron under both magnetic and electric fields**

Check-point 10 – Q1

The magnetic force acting on a charge moving in a direction perpendicular to the magnetic field increases when

(1) the charge ↑

(2) the charge is moving faster

(3) a stronger magnet is used

A (1) and (2) only

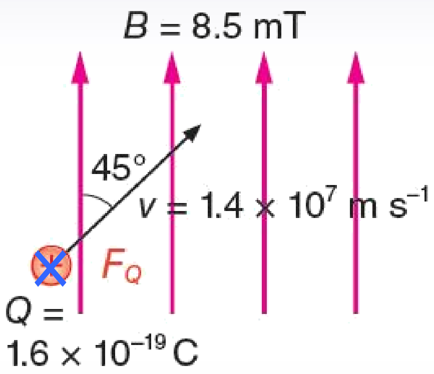
B (1) and (3) only

C (2) and (3) only

D (1), (2) and (3)

Check-point 10 – Q2

A proton (1.6×10^{-19} C), travelling at 1.40×10^{7} m s^{-1}, flies into a magnetic field of 8.5 mT at 45°.



Find the magnitude of the magnetic force and indicate its direction at the moment it enters the magnetic field.

$$F_Q = BQv \sin \theta$$

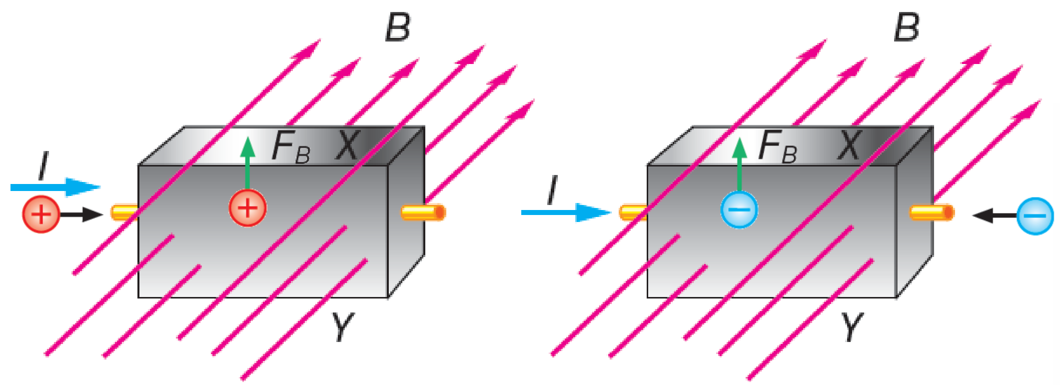
$$= (8.5 \times 10^{-3})(1.6 \times 10^{-19})(1.4 \times 10^{7})(\sin 45°)$$

$$= 1.35 \times 10^{-14} \text{ N}$$

4 Hall voltage

a Explanation of Hall effect

A current-carrying conductor in a magnetic field:



Charge carriers experience an upward magnetic force F_B.

Charges accumulated on X.

⇒ A p.d. (**Hall voltage**) develops across XY

⇒ **Hall effect**

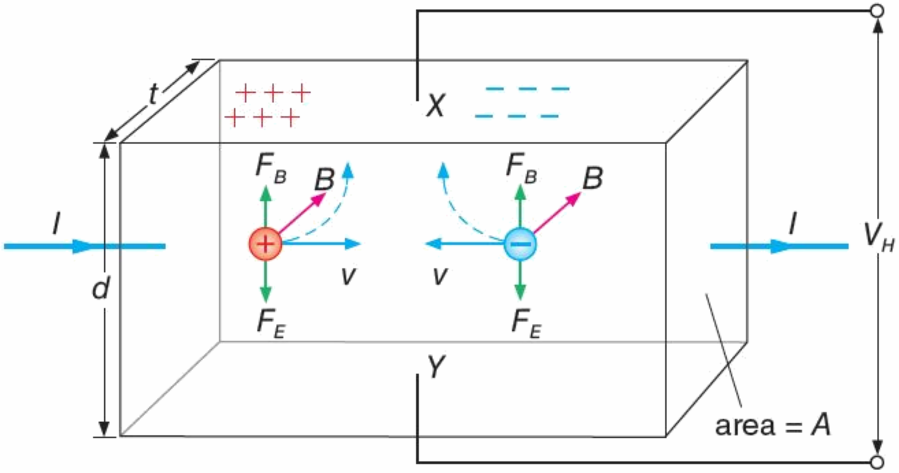
4 Hall voltage

b Derivation of Hall voltage

Electric field across *XY* is due to the accumulated charges on one side of the conductor by Hall effect.

Accumulation continues until electric force F_E = magnetic force F_B

$\Rightarrow$ steady Hall voltage V_H



Charges keep on flowing through the conductor with drift velocity v, forming current I.

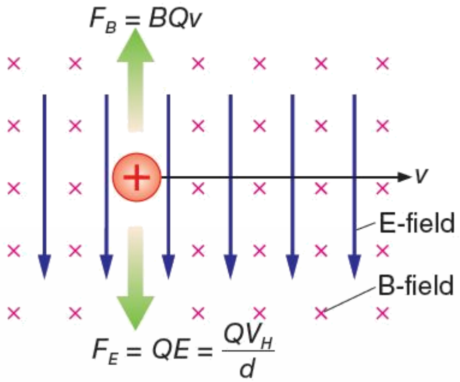
b Derivation of Hall voltage

When electric force balances magnetic force,

$$F_E = F_B$$

$$QE = BQv$$

$$E = Bv \text{ .... (1)}$$



Surfaces X and Y act like two parallel plates with separation d and p.d. V_H.

$$\Rightarrow E = \frac{V_H}{d} \Rightarrow \text{(1) becomes } \frac{V_H}{d} = Bv$$

$$\Rightarrow V_H = Bvd \text{ ....(2)}$$

F-X Ext

b Derivation of Hall voltage

By the relation between current and drift velocity,

$$I = nAvQ \text{..................................} (3)$$

Re-arrange the terms in (3) and by $A = td$,

$$v = \frac{I}{ntdQ} \text{ ....} (4)$$

Substitute (4) into $V_H = Bvd$, $V_H = B\frac{I}{ntdQ}d$

$\Rightarrow$ Hall voltage: $$\boxed{V_H = \frac{BI}{nQt}}$$

4 Hall voltage

c Characteristics of conductors revealed by Hall voltage

Metals have –ve charge carriers (e^-); semiconductors have +ve or –ve charge carriers.

Direction of F_B depends only on current direction, not the sign of charge carriers

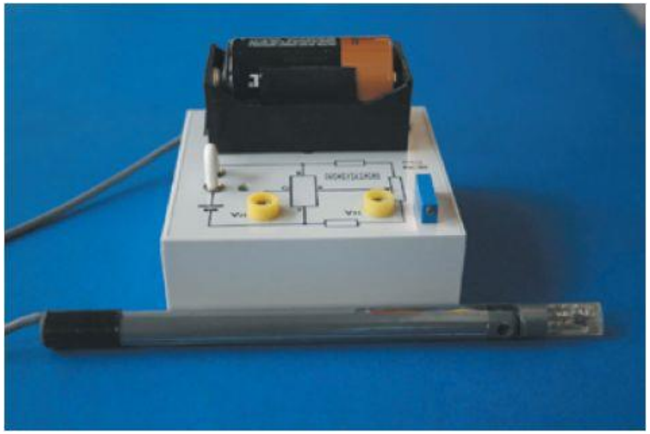
⇒ the latter is determined by the sign of Hall voltage.

Hall voltage measures both magnetic field and charge carrier density of a conductor.

Example 17 **Finding the number of charge carriers and drift velocity**

5 Measuring magnetic fields by a Hall probe

Hall probe



- measures the strength of steady B-field based on Hall voltage.
- consists of a circuit box and a probe head.

The battery in circuit box provides a steady I through a semiconductor in the probe head. The semiconductor has a constant thickness t and contsant density of charge carriers n.

5 Measuring magnetic fields by a Hall probe

By $V_H = \dfrac{BI}{nQt}$, Hall voltage $V_H \propto$ B-field B

∵ voltmeter shows the magnitude of B-field

∴ Hall probe compares relative strengths of B-fields directly

However, voltmeter needs to be first calibrated.

Expt 4i Measuring magnetic fields with Hall probe

5 Measuring magnetic fields by a Hall probe

Example 18 **Hall voltage in different materials**

Check-point 11 – Q1–2

True or False:

1. Hall voltage is proportional to the magnetic field.

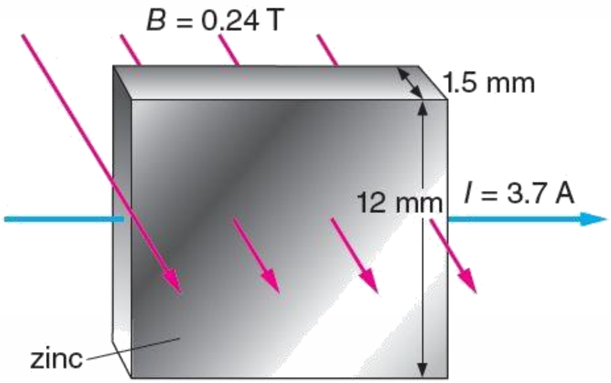
   (T / F)

2. A Hall probe is used to measure a changing magnetic field.

   (T / F)

Check-point 11 – Q3

A strip of zinc is 12 mm wide and 1.5 mm thick, has 19×10^{28} charge carriers per unit volume, carries 3.7-A current.



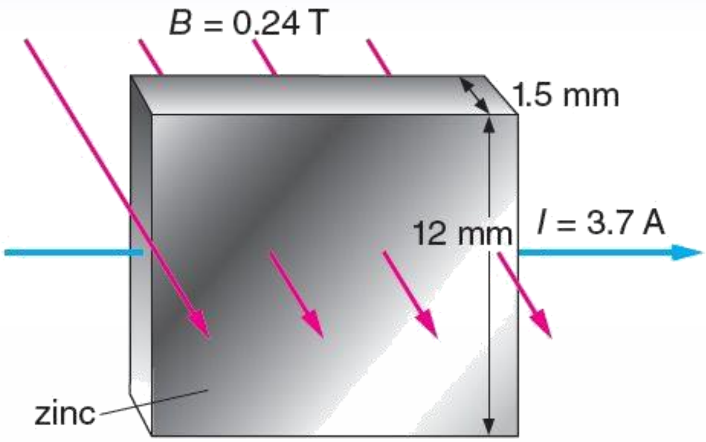
A uniform B-field of 0.24 T passes through it at right angle.
(Charge of each charge carrier = 1.6×10^{-19} C)

(a) Find the Hall voltage across the strip.

$$V_H = \frac{BI}{nQt} = \frac{(0.24)(3.7)}{(19 \times 10^{28})(1.6 \times 10^{-19})(1.5 \times 10^{-3})}$$

$$= 1.95 \times 10^{-8}\ \text{V}$$

Check-point 11 – Q3



(b) Find the drift velocity of electrons inside.

$$v = \frac{E}{B} = \frac{V_H}{Bd} = \frac{1.95 \times 10^{-8}}{0.24 \times 12 \times 10^{-3}}$$

$$= 6.77 \times 10^{-6} \text{ m s}^{-1}$$

5 Measuring magnetic fields by a Hall probe

Example 19 **Circular motion of an electron beam**

The End